

Black hole as topological insulator

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Black holes are extraordinary massive objects which can be described classically by general relativity, and topological insulators are new phases of matter that could be use to built a topological quantum computer. They seem to be different objects, but in this paper, we claim that the black hole can be considered as a kind of topological insulator. For BTZ black hole in three dimensional AdS_3 spacetime we give two evidences to support this claim: the first evidence comes from the black hole “membrane paradigm”, which says that the horizon of black hole behaves like an electrical conductor. On the other hand, the vacuum can be considered as an insulator. The second evidence comes from the fact that the horizon of BTZ black hole can support two chiral massless scalar field with opposite chirality. Those are two key properties of 2D topological insulator. For higher dimensional black hole the first evidence is still valid. So we conjecture that the higher dimensional black hole can also be considered as higher dimensional topological insulators. This conjecture will have far-reaching influences on our understanding of quantum black hole and the nature of gravity.

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I. INTRODUCTION

Black holes are massive objects. They have so strong gravitational field that even the light cannot escape. Their existence in universe is supported by many observational evidences, the most direct one comes from the gravitational waves [1]. Even though the black holes are solutions of classical general relativity, the quantum effect cannot be neglected near the horizon. Indeed the seminal work of Bekenstein [2] and Hawking [3] show that the black holes are actually perfect black body which has temperature and entropy. Up to now, we still don't have a complete quantum theory of black hole.

The black hole “membrane paradigm” [4, 5] says that to an outside observer, the horizon behaves like a viscous fluid. Besides, it is also electrical conductive. On the other hand, the vacuum inside the horizon can be considered as insulator. This property is similar to the topological insulator.

Topological insulators [6, 7] are new orders of matter. Unlike the usual order which are associated with a broken symmetry, this new order has topological origin, and can be described by topological field theory [8]. Roughly speaking, a topological insulator is a bulk insulator but has conducting boundary states. They exist both in two [9–11] and three dimensions [12–14]. It was shown [15] that BF theory is the effective theory for topological insulators in 2D and 3D just as the Chern-Simons theory for the quantum Hall effect [16, 17]. This BF theory can describe key properties of topological insulators, including the elementary excitation and their statistics, the edge theory and the importance of the T-symmetry.

In this paper, we show that the black hole can be considered as topological insulator. The paper is organized as follows. In section 2, the BTZ black hole in AdS_3 is analyzed. We give two evidences to support the claim that BTZ black hole can be considered as topological insulators. In section 3, we conjecture that higher dimensional black holes also can be considered as higher dimensional topological insulators. Section 4 is the conclusion.

II. BTZ BLACK HOLE

In three dimensional spacetime, things become easier and more clear. When there exists a negative cosmology constant $\Lambda < 0$, the black hole solution—which is called BTZ black hole [18]—can exist. Due to the membrane paradigm [19], the conductivity of the horizon is

$$\sigma = \frac{1}{g^2}, \quad (1)$$

where g is the gauge coupling constant. Thus the horizon can be considered as metallic and the vacuum as insulator. This is just the salient property of 2D topological insulator.

Another property of topological insulator is that they have two chiral bosonic modes flowing in opposite directions. Those boundary modes appear because the boundary breaks the gauge symmetry. Those boundary states are also appeared in gravity theory, which was studied long time ago [20, 21]. In Ref.[22], it was shown that the horizon of BTZ black hole can support a pair of chiral massless scalar fields with opposite chirality which is the same as topological insulator.

In appendix we give another derivation for this result which is familiar for condensed matter physicist [15, 16]. One can get this result in three steps: Firstly, as shown in Ref.[23, 24], $(2 + 1)$ -dimensional general relativity with $\Lambda < 0$

can be cast into $SO(2, 1) \times SO(2, 1)$ Chern-Simons theory.

$$I_{GR}[e, \omega] = \frac{1}{8\pi G} \int e^a \wedge (d\omega_a + \frac{1}{2}\epsilon_{abc}\omega^b \wedge \omega^c) - \frac{1}{6L^2}\epsilon_{abc}e^a \wedge e^b \wedge e^c$$

$$= I_{CS}[A^{(+)}] - I_{CS}[A^{(-)}], \quad (2)$$

where e^a and ω^a are the co-triad and spin connection 1-form respectively, $A^{(\pm)a} = \omega^a \pm \frac{1}{L}e^a$. Secondly, it is well known that on a manifold with boundary, the Chern-Simons theory reduces to a chiral Wess-Zumino-Novikov-Witten(WZNW) theory on the boundary [25], thus

$$I_{CS}[A] = I_{WZNW}[g], \quad (3)$$

where the $SO(2, 1)$ group element g is defined through $A_i = g^{-1}\partial_i g$.

The final step is to consider the horizon as boundary. Due to the horizon boundary condition, the action of chiral WZNW theory reduces to the action of chiral massless scalar field theory. Thus

$$I_{WZNW}[g] = I_{Scalar}[\Psi], \quad (4)$$

where Ψ is a component of Gauss decomposition of the g , which depend only on $\tilde{u} = \varphi + \frac{v}{L}$. So it is a chiral massless scalar field.

Since general relativity have two copy of Chern-Simons theories, it contain two chiral massless scalar field with opposite chirality. This is the same as the 2D topological insulator [15].

Combined with the “membrane paradigm”, we can confirm that BTZ black hole can be considered as 2D topological insulator.

III. HIGHER DIMENSIONAL BLACK HOLES

In higher dimensional spacetime ($D \geq 4$), general relativity can't be reformulated as CS theory, so we can't follow the process in the last section. But the membrane paradigm is still valid, thus the horizon can conduct electrical current. For example, in four dimension, the surface electrical resistivity is

$$\rho = 4\pi\alpha\hbar/e^2 = 2\alpha R_H, \quad (5)$$

where α is the finer structure constant and R_H is the Hall resistivity. And the vacuum is also an insulator, so this suggest that higher dimensional black holes can also be considered as topological insulators.

In higher dimension, the general relativity can be reformulated as constraint BF theory [26, 27]. It was shown that there are gauge degrees of freedom on the boundary [21]. But it is still unclear what is the effective theory to describe those degrees of freedom. On the other hand, the effective theory for topological insulators is abelian BF theory, which can give the correct boundary theory for topological insulators [15].

IV. CONCLUSION

In this paper, we claim that black holes can be considered as topological insulators. For BTZ black hole in AdS_3 spacetime, we give two evidences to support this claim. The first comes from the black hole membrane paradigm, and

the second comes from the fact that on the horizon there exist two chiral massless scalar field with opposite chirality. The first evidence is also valid for higher dimensional black holes.

If this claim is true for all kind of black holes, it will have far-reaching influence on our understanding of quantum black hole and the nature of gravity. Since we have well understanding on the topological insulator, including the band structure, the lattice models, the modified Dirac equation description and so on [28]. Those properties can be translated to the quantum black hole. Also since the topological insulator is a new phase of matter, the formation of black hole can be considered as a topological phase transition without symmetry breaking.

General relativity can be formulated as constraint BF theory, and the effective theory of 2D and 3D topological insulators is also BF theory. This suggest that maybe the gravity is also an effective theory of some more fundamental objects. Thus gravity is an “emergent” phenomenon [29–32].

Appendix A: The field theory on the horizon of BTZ black hole

For the case of negative cosmology constant $\Lambda = -1/L^2$, one can define two $SO(2, 1)$ connection 1-form

$$A^{(\pm)a} = \omega^a \pm \frac{1}{L}e^a, \quad (A1)$$

where e^a and ω^a are the co-triad and spin connection 1-form respectively. Then up to boundary term, the first order action of gravity can be rewritten as

$$\begin{aligned} I_{GR} &= \frac{1}{8\pi G} \int e^a \wedge (d\omega_a + \frac{1}{2}\epsilon_{abc}\omega^b \wedge \omega^c) - \frac{1}{6L^2}\epsilon_{abc}e^a \wedge e^b \wedge e^c \\ &= I_{CS}[A^{(+)}] - I_{CS}[A^{(-)}], \end{aligned} \quad (A2)$$

where $A^{(\pm)} = A^{(\pm)a}T_a$ are $SO(2, 1)$ gauge potential, and the Chern-Simons action is

$$I_{CS} = \frac{k}{4\pi} \int Tr\{A \wedge dA + \frac{2}{3}A \wedge A \wedge A\}, \quad (A3)$$

with

$$k = \frac{L}{4G}. \quad (A4)$$

One can get the CS equation

$$F^\pm = dA^{(\pm)} + A^{(\pm)} \wedge A^{(\pm)} = 0. \quad (A5)$$

The equation implies that the potential A can be locally written as

$$A = g^{-1}dg. \quad (A6)$$

To study the field theory on the horizon of BTZ black hole, it is more appropriate to adopt the advanced Eddington coordinate. The metric of BTZ black hole is

$$ds^2 = -N^2 dv^2 + 2dvdr + r^2(d\varphi + N^\varphi dv)^2. \quad (A7)$$

Choose the following co-triads

$$l_a = -\frac{1}{2}N^2 dv + dr, \quad n_a = -dv, \quad m_a = rN^\varphi dv + rd\varphi, \quad (A8)$$

which gives the following connection:

$$A^{-(\pm)} = -(N^\varphi \mp \frac{1}{L})dr - \frac{N^2}{2}d(\varphi \pm \frac{v}{L}), \quad A^{+(\pm)} = -d(\varphi \pm \frac{v}{L}), \quad A^{2(\pm)} = r(N^\varphi \pm \frac{1}{L})d(\varphi \pm \frac{v}{L}). \quad (\text{A9})$$

Define new variables which is useful later,

$$u = \varphi - \frac{v}{L}, \quad \tilde{u} = \varphi + \frac{v}{L}. \quad (\text{A10})$$

A crucial property of the connection is that, on the whole manifold, one has

$$A_u^{(+)} \equiv 0, \quad A_{\tilde{u}}^{(-)} \equiv 0. \quad (\text{A11})$$

On a manifold with the form $M = R \times \Sigma$, the Chern-simons action can be written canonically as

$$I_{CS} = -\frac{k}{4\pi} \int dx^0 \int_\Sigma \varepsilon^{ij} Tr(A_i \dot{A}_j - A_0 F_{ij}). \quad (\text{A12})$$

For the right section $A^{(+)}$, choose u as time coordinate and r, \tilde{u} as the other coordinates. Due to condition (A11), $A_0^{(+)} \equiv 0$ gives the constrain

$$F_{ij} = 0. \quad (\text{A13})$$

This constrain can be solved by $A_i = g^{-1} \partial_i g$. Submit this solution into the action (A12) one can get

$$I_{CS} = \frac{k}{4\pi} \int_{\partial M} Tr(g^{-1} \partial_u g g^{-1} \partial_{\tilde{u}} g) + \frac{1}{12\pi} \int_M Tr(g^{-1} dg)^3, \quad (\text{A14})$$

which is just the chiral WZNW action. This is valid for arbitrary boundary. Next we pay attention to the horizon of BTZ black hole.

For a general $SO(2, 1)$ group element $g(\tilde{u}, u, r)$, using the Gauss decomposition, it can be written as

$$g = \begin{pmatrix} 1 & \frac{1}{\sqrt{2}}x_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-\Psi_1/2} & 0 \\ 0 & e^{-\Psi_1/2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{\sqrt{2}}y_1 & 1 \end{pmatrix}. \quad (\text{A15})$$

Within this parameter, the WZNW action is [33]

$$kI_{WZNW} = \frac{k}{4\pi} \int_{\partial M} dud\tilde{u} \frac{1}{2} (\partial_u \Psi \partial_{\tilde{u}} \Psi - e^\Psi (\partial_u x \partial_{\tilde{u}} y + \partial_u y \partial_{\tilde{u}} x)). \quad (\text{A16})$$

Let's solve the constrain equation (A13) near the horizon. The property of horizon is encode in $N^2 = 0$. Near the horizon $r = r_+$, a small parameter $\epsilon = r - r_+$ can be defined, and $N^2 \approx 2\kappa\epsilon$, so

$$A_{\tilde{u}}^- \approx -\kappa\epsilon. \quad (\text{A17})$$

On the other hand,

$$A_{\tilde{u}}^- = e^{\Psi_1} \partial_{\tilde{u}} x_1 / \sqrt{2}, \quad (\text{A18})$$

and Ψ_1 is finite at horizon, so one have

$$x_1 \sim \epsilon, \quad (\text{A19})$$

which vanish on the horizon. So the final action on the horizon is

$$\begin{aligned} kI_{WZNW} &= \frac{k}{4\pi} \int_{\partial M} dud\tilde{u} \frac{1}{2} \partial_u \Psi_1 \partial_{\tilde{u}} \Psi_1 \\ &= \frac{k}{4\pi L} \int_{\partial M} d\varphi dv [(\partial_v \Psi_1)^2 - L^2 (\partial_\varphi \Psi_1)^2], \end{aligned} \quad (\text{A20})$$

with Ψ_1 depending only on $\tilde{u} = \varphi + \frac{v}{L}$. So it is an action for chiral massless scalar field Ψ_1 .

The similar results can be get for the $A^{(-)}$, which gives another chiral massless scalar field Ψ_2 depending only on u . Thus, on the horizon there exist two chiral massless scalar field with opposite chirality, the same as topological insulator.

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